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Form factors and charge radii of light and heavy flavoured mesons in a QCD inspired quark model with two loop static potential

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Abstract We incorporate the effects of two loop static potential in the results of form factors and charge radii of both light and heavy flavoured pseudo scalar mesons in a QCD inspired quark model pursued by us For heavy light mesons we also report the results in the infinite heavy quark mass limit

Keywords Two loop static potential form factors charge radii

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1. Introduction

The study of meson wave functions and their phenomenology is an important topic in QCD [1] as one of the basic ingredients of the static and dynamical properties of a hadron is its wave function

Meson wave functions are in principle calculable from simple QCD potentials within Schrödinger equation approach if they are less singular than $1/r^2$ [2, 3] Furthermore, the application of QCD potentials to light quark systems is handicapped due to their ad-hoc relativistic modifications [4] Any QCD potential based approach must take into account the above limitations

Some time back [5, 6], such a specific QCD Inspired quark model has been proposed by us The model uses the two body Schrodinger equation and obtains the first order perturbed wave function of the QCD potential using Dalgarno's method [7] In order to test the model wave function, we have calculated [5] the properties of light and heavy flavoured mesons such as form factors, charge radii, masses and decay constants both in finite

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and infinite heavy quark mass limit [6], as well as the Isgur-Wise function of heavy light mesons [8]

Recent years have however seen unprecedented progress in the study of QCD potentials Explicit calculations have been performed to obtain two loop static QCD potentials pioneered by Peter [9] and Schröder [10, 11].

In this paper, we reinvestigate the results of [5, 6] incorporating the two loop effects on the form factors and charge radii of mesons both in the finite and infinite heavy quark mass limit In section 2, we discuss the formalism, while in section 3, we summarise the results Section 4 contains the summary and conclusion

2. Formalism

2 1 The Model :

The essential features of the model have been reported in reference [5] We summarise them for completeness With the spin independent Fermi-Breit Hamiltonian with three colour degree of freedom

$$H = -(4\alpha_S / 3r) + br + c \tag{1}$$

and taking $H' = br + c$ (2)

as perturbation, one obtains the relativised normalized wave function with confinement parameter b to be

$$\Psi_{rel+conf}(r) = \left(N' / \sqrt{\pi a_0^3}\right)\left(1 - \tfrac{1}{2}\mu b a_0 r^2\right)(r / a_0)^{-\varepsilon} e^{-\frac{r}{a_0}} \tag{3}$$

with the normalization constant

$$N' = 2^{\frac{1}{2}} / \left(\sqrt{2^{2\varepsilon}\left\{\Gamma(3-2\varepsilon) - \frac{1}{4}\mu b a_0^3 \Gamma(5-2\varepsilon) + \frac{1}{64}\mu^2 b^2 a_0^6 \Gamma(7-2\varepsilon)\right\}}\right) \tag{4}$$

where $\mu = m_q m_Q / (m_q + m_Q)$ (5)

is the reduced mass, and $a_0 = (4\mu\alpha_S / 3)^{-1}$ (6)

and $\varepsilon = 1 - (1 - 4\alpha_S / 3)^{-1}$ (7)

2.2. Two Loop Static Potential :

The static two loop potential is conveniently defined in V scheme as

$$V(r) = -C_F \alpha_V \left(1 / r^2\right) / r \tag{8}$$

Here α_V represents the effective coupling constant which incorporates the entire radiative corrections into its definition and

$$C_F = (N_C^2 - 1)/2N_C \tag{9}$$

Traditionally, the strength of the quark gluon interaction is characterized by the coupling constant $\alpha_{\overline{MS}}(q^2)$ defined in $\overline{MS}$ regularization scheme V scheme is a physically based alternative to this usual scheme. The relationship of $\alpha_V\left(1/r^2\right)$ to the conventional coupling constant in MS scheme at the two loop level has been reported by Peter [9] and later Schröder [11] has corrected the error in the gluon sector Incorporating these corrections, the relationship between α_V and $\alpha_{\overline{MS}}$ at the NNLO level is

$$\alpha_V\left(1/r^2\right) = \alpha_{\overline{MS}}\left\{1 + (\alpha_{MS}/4\pi)(a_1 + \beta_0 L) + (\alpha_{MS}/4\pi)^2\right.$$

$$\left.\left(a_2 + \beta_0^2\right)\left(L^2 + \pi^2/3\right) + (\beta_1 + 2\beta_0 a_1)L) + O\left(\alpha_{MS}^3\right)\right\} \tag{10}$$

where $$L = 2\ln(\tilde{\mu} r \exp\gamma) \tag{11}$$

and $$\beta_0 = \tfrac{11}{3}C_A - \tfrac{4}{3}T_F n_f \tag{12}$$

and $$\beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F n_f - \frac{20}{3}C_A T_F n_F \tag{13}$$

are the first two terms of the beta function, $\tilde{\mu}$ being the scale parameter, a_1 and a_2 being the one loop and two loop constants. The scale parameter are judiciously chosen in order to reduce large logarithmic corrections. There are several choices such as :

i) The first choice $\tilde{\mu} = 1/r$ which eliminates r dependence in L altogether,

ii) the second choice $\tilde{\mu} = \exp(-\gamma_E - a_1/2\beta_0)/r$ eliminates the one loop coefficient completely ; and the

iii) and the third choice $\tilde{\mu} = exp(-\gamma_E)/r^2$ removes all terms involving the Euler constant γ_E from the coefficients

2.3. Form Factor and Charge Radii :

The elastic charge form factor for a charged system of point quarks has the form [12,13]

$$eF\left(Q^2\right) = \sum_{i=1}^{2}(e_i/Q_i)\int^{\infty} r|\Psi(r)|^2 \sin(Q_i r)\,dr \tag{14}$$

where e_i is the charge of the i^{th} quark/antiquark and

$$Q_i = \sum_{j \neq i} m_j Q / \sum_{i=1}^{2} m_i \tag{15}$$

Q_i describes how the virtuality Q is shared between the quark antiquark pair of the meson. Then, using eq. (3) in eq. (14) with improved approximations in the series expansion of $(Q_i r)$ and integrating over r,

$$eF(Q^2) = \frac{N_1^2}{a_0 2^{-2\varepsilon}} \Sigma_1^2 e_i \times$$

$$\Big[\left(1 + a_0^2 Q_i^2/4\right)^{\varepsilon - 1} \left\{ (2-2\varepsilon) X_i - \frac{1}{6}(2-2\varepsilon)^3 Q_i^2 X_i^3 + \frac{1}{120}(2-2\varepsilon)^5 Q_i^4 X_i^5 \right\} \Gamma(2-2\varepsilon)$$
$$- \frac{1}{4}\mu b a_0^2 \left(1 + a_0^2 Q_i^2/4\right)^{\varepsilon - 2} \left\{ (4-2\varepsilon) X_i - \frac{1}{6}(4-2\varepsilon)^3 Q_i^2 X_i^3 + \frac{1}{120}(4-2\varepsilon)^5 Q_i^4 X_i^5 \right\} \Gamma(4-2\varepsilon)$$
$$+ \frac{1}{64}\mu^2 b^2 a_0^6 \left(1 + a_0^2 Q_i^2/4\right)^{\varepsilon - 3} \left\{ (6-2\varepsilon) X_i - \frac{1}{6}(6-2\varepsilon)^3 Q_i^2 X_i^3 + \frac{1}{120}(6-2\varepsilon)^5 Q_i^4 X_i^5 \right\} \Gamma(6-2\varepsilon) \Big] \tag{16}$$

where $$X_i = \frac{Q_i^4}{120}\left(\frac{4}{a_0^2} + Q_i^2\right)^{-\frac{5}{2}} + \frac{Q_i^2}{6}\left(\frac{4}{a_0^2} + Q_i^2\right)^{-\frac{3}{2}} + \left(\frac{4}{a_0^2} + Q_i^2\right)^{-\frac{1}{2}} \tag{17}$$

and N_1 is the normalization constant.

In the V-scheme, a_0 and ε as defined in eqs. (6) and (7) should read as

$$a_0^V = (4\mu\alpha_V/3)^{-1} \tag{18}$$

and

$$\varepsilon_V = (1 - 4\alpha_V/3)^{-1}. \tag{19}$$

The average charge radii square for the mesons are obtained from the relation

$$\langle r^2 \rangle = -6 \frac{d}{dQ^2} eF(Q^2)_{Q^2=0} \tag{20}$$

Using eq. (16) in eq. (20), we obtain the average charge radii square in V scheme for the mesons having quark masses m_i and m_j and charges e_i and e_j respectively,

$$\langle r^2 \rangle = a_0^{V2}\left[e_i\left(1+\frac{m_i}{m_j}\right)^{-2} + e_j\left(1+\frac{m_j}{m_i}\right)^{-2} \right] \times$$

$$\frac{\left[\begin{array}{l} \Gamma(2-2\varepsilon_V)\left\{\frac{(2-2\varepsilon_V)}{2}+\frac{(2-2\varepsilon_V)^3}{4}+\frac{3(2-\varepsilon_V)(\varepsilon_V-1)}{2}\right\} \\ -\frac{1}{4}\mu b a_0^{V3}\Gamma(4-2\varepsilon_V)\left\{\frac{(4-2\varepsilon_V)}{2}+\frac{(4-2\varepsilon_V)^3}{4}+\frac{3(4-2\varepsilon_V)(\varepsilon_V-2)}{8}\right\} \\ +\frac{1}{64}\mu^2 b^2 a_0^{V6}\Gamma(6-2\varepsilon_V)\left\{\frac{(6-2\varepsilon_V)}{2}+\frac{(6-2\varepsilon_V)^3}{4}+\frac{3(6-2\varepsilon_V)(\varepsilon_V-3)}{128}\right\} \end{array}\right]}{\left[\begin{array}{l} (2-2\varepsilon_V)\Gamma(2-2\varepsilon_V)-\frac{1}{4}\mu b a_0^{V3}(4-2\varepsilon_V)\Gamma(4-2\varepsilon_V)+ \\ \frac{1}{64}\mu^2 b^2 a_0^{V6}(6-2\varepsilon_V)\Gamma(6-2\varepsilon_V) \end{array}\right]} \quad (21)$$

2.4. Infinite Heavy Quark Mass Limit :

In the infinite heavy quark mass limit, $m_Q \to \infty$ and the reduced mass μ reduces to m_q and consequently a_0^V change to $a_0^{V\infty}$,

$$a_0^{V\infty} = \left(4 m_q \alpha_V / 3\right)^{-1} \quad (22)$$

These changes are incorporated in eqs. (16) and (21).

3. Results

3.1. Relationship between $\alpha_{\overline{MS}}$ and α_V :

Taking $n_f = 4$ and $n_f = 5$ and the corresponding value of $\alpha_{\overline{MS}}$ at the scale of the c- and b- quark mass $\alpha_{\overline{MS}}(m_c) = 0.39, n_f = 4$ and $\alpha_{M\bar{S}}(m_b) = 0.22, n_f = 5$, [14] respectively, the value of $\alpha_V(1/r^2)$ for the three choices of $\tilde{\mu}$ have been evaluated earlier [15]. It shows that the two loop potential invariably scales up the effective coupling constant.

3.2. Form factors :

We have calculated form factors and charge radii of light and heavy flavoured mesons with two loop potential within a QCD Inspired quark model using the well known QCD factors, $N_C = 3$, $C_F = 4/3, T_F = 1/2, C_A = 3$ along with the numerical value $\rho(3) = 1.202$ and $\gamma_E = 0.5772$ .We have also improved our earlier calculation at leading order level [5].

Table 1 presents the numerical values of form factors of pions with two loop effects and compares with experimental data. Table 2 presents the data comparison for kaons. The tables for both pions and kaons show that data seems to favour $b = 0$.

Even though there is no experimental information available for form factors of mesons having charm and bottom quarks, we plot the results for form factors of D mesons in Figures 1 and for B mesons in Figures 2 for $b = 0$ and $b = 0.01\,\mathrm{GeV}^2$ with $\alpha_{\overline{MS}}(m_c) = 0.39, n_f = 4$ for D mesons and with $\alpha_{\overline{MS}}(m_b) = 0.22, n_f = 5$ for B mesons respectively (LO lines); with $\alpha_V = 0.65$ for D mesons and with $\alpha_V = 0.26$ for B mesons (NLO lines). These figures show that two loop effects (NLO) and those without such effects (LO) are widely different for $b = 0.01\,\mathrm{GeV}^2$, but the change is insignificant for $b = 0$ for both D and B mesons. Figures 1 and 2 also display the results of infinite heavy quark mass limit (NLO(inf) and LO(inf) lines). Effect of such limit is sensitive to both Q^2 and the confinement parameter b.

Table 1. Form Factors of pions with $\alpha_V = 0.65$.

$Q^2(GeV/c)^2$	$\lvert F_\pi\rvert^2$ [14]	$\lvert F_\pi\rvert^2 (b=0)$	$\lvert F_\pi\rvert^2 (b=0.01GeV^2)$
0. 015	0.944 ±0.007	0.934	0.866
0.027	0.898± 0.008	0.886	0.792
0.037	0.876± 0.011	0.848	0.745
0.050	0.830± 0.010	0.803	0.699
0.101	0.680± 0.017	0.658	0.605
0.125	0.665± 0 023	0.603	0.583
0.203	0.529±0.040	0.466	0.532
0.253	0.336±0.073	0.402	0.502

Table 2. Form Factors of Kaons with $\alpha_V = 0.65$.

$Q^2(GeV/c)^2$	$\lvert F_K\rvert^2$ [15]	$\lvert F_\pi\rvert^2 (b=0)$	$\lvert F_\pi\rvert^2 (b=0.01GeV^2)$
0.0175	0.965± .024	0.936	0.930
0.0275	0.973± .031	0.903	0.899
0.0375	0.918± .043	0.871	0.872
0.0475	0.877± .055	0.841	0.850
0.0575	0.880±0.071	0.813	0.829
0.0675	0.742±0 088	0.786	0.811
0.0775	0.720± 0.120	0.760	0.795
0.0850	0.730± 0.010	0.742	0.784
0.0950	0.800±0.140	0.719	0.770

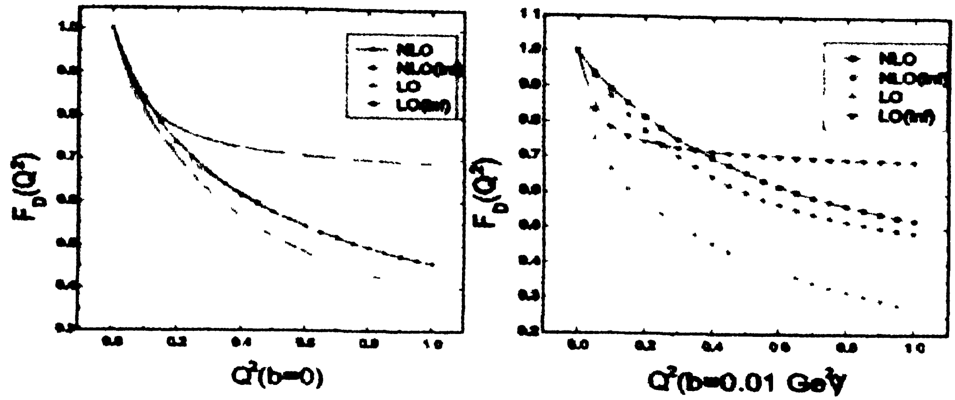

Figure 1 $F_D(Q^2)$ *vs* Q^2 for D mesons

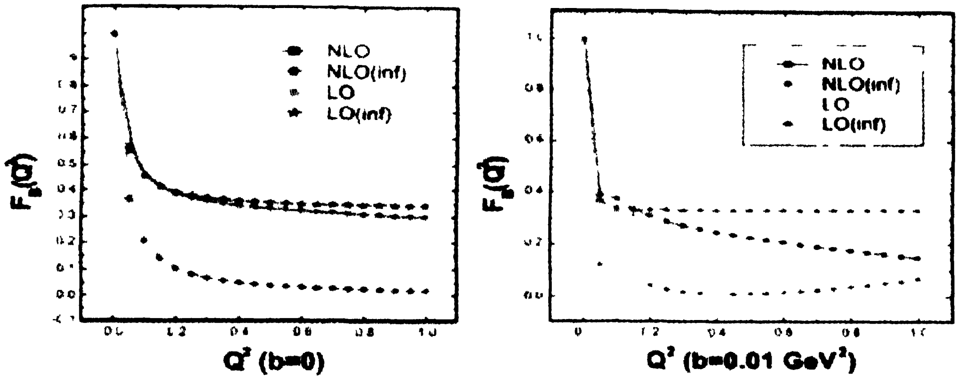

Figure 2 $F_B(Q^2)$ *vs* Q^2 for B mesons

3 3 *Charge Radii*

We present the results obtained for the charge radii for pions and kaons in Table 3 and compare with the data as well as with the predictions of the light front quark model [16] using b = 0 and b = 0 01GeV^2 Data favours b = 0 The mean square charge radii of the heavy pseudo scalar mesons have not been measured yet We present the results of our calculations of their charge radii square both for finite heavy quark masses as well in the infinite mass limit in Table 4 (with b = 0) and compare our results with those of

Table 3. The Mean square charge radii of pions and kaons with α_V = 0 65

	π^+	K^+	K^0
This work, b=0	0 370	0 309	-0 064
This work, b=0 01GeV^2	1 025	0 440	-0 092
Hwang [16]	0 443	0 349	-0 0676
Burden *et al* [19]	0 314	0 240	-0 020
Expt	0 439±0 008[17]	0 34±0 050[18]	-0 054± 0 026[20]

Hwang [16]. The results for the D mesons are in good agreement, but for B mesons, they differ significantly.

Table 4. The mean square charge radii of D ($\alpha_V = 0.65$) & B mesons ($\alpha_V = 0.26$).

Meson	$\langle r^2 \rangle_F$, $(b=0)$	$\langle r^2 \rangle_-$, $(b=0.01GeV^2)$	$\langle r^2 \rangle_F$, [16]	$\langle r^2 \rangle_-$, [16]
D^+	0.134	0.182	0.184	0.248
D^0	-0.234	-0.363	-0.304	-0.496
D_0^+	0.126	0.182	0.124	0.181
B^+	2.960	3.371	0.378	0 496
B^0	-1.471	-1.680	-0.187	-0.248
B_S^0	-1.373	-1.681	-0.119	-0.181
B_C^+	2.050	3.372	0.043	–

4. Conclusion

We have calculated form factors and charge radii of light and heavy flavoured mesons with two loop potential within a QCD inspired quark model .We have improved our earlier calculation at the leading order level. The predictions are consistent with current experimental data as well as with those of other models except for charge radii of B mesons.

The earlier analyses [5, 6] revealed two limitations of the QCD Inspired quark model:

Firstly, in the earlier work, calculated results were in good agreement with other theoretical predictions and experimental data when the running coupling constant was frozen at around $\alpha_V = 0.6$ to 0.65, in contrast to $\alpha_{\overline{MS}} = 0.39$ for D mesons and $\alpha_{MS} = 0.22$ for B mesons. Two loop calculations raise the value of the running coupling constant from $\alpha_{\overline{MS}} = 0.39$ for D mesons to $\alpha_V = 0.65$, and justify setting $\alpha = 0.65$ to obtain agreement with other theoretical work [16] . For B mesons however, two loop calculations raise $\alpha_{\overline{MS}} = 0.22$ to $\alpha_V = 0.26$ which falls short of the desired value of 0.6.

Secondly, whereas spectroscopic analysis places the value of the confinement parameter b to be equal to 0.183GeV^2 , the earlier work shows that model predictions fit best with data for $b = 0$.In the present work too, the same feature remains. A larger value of b cannot be incorporated in the model.

The model presumably needs reformulation so that significant confinement effects can be incorporated and also so as to be able to justify obtaining good results for B mesons with $\alpha_V = 0.6$.

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